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Origami world

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Abstract: We paste together patches of AdS_6 to find solutions which describe two 4-branes intersecting on a 3-brane with non-zero tension. We construct explicitly brane arrays with Minkowski, de Sitter and Anti-de Sitter geometries intrinsic to the 3-brane, and describe how to generalize these solutions to the case of AdS_{4+n}, $n > 2$, where n $n+2$-branes intersect on a 3-brane. The Minkowski and de Sitter solutions localize gravity to the intersection, leading to 4D newtonian gravity at large distances. We show this explicitly in the case of Minkowski origami by finding the zero-mode graviton, and computing the couplings of the bulk gravitons to the matter on the intersection. In de Sitter case, this follows from the finiteness of the bulk volume. The effective 4D Planck scale depends on the square of the fundamental 6D Planck scale, the AdS_6 radius and the angles between the 4-branes and the radial AdS direction, and for the Minkowski origami it is ${M_4}^2 = \frac{2}{3}(\tan\alpha_1 + \tan\alpha_2)\, {M_*}^4 L^2$. If $M_* \sim$ few $\times$ TeV this may account for the Planck-electroweak hierarchy even if $L \sim 10^{-4}$m, with a possibility for sub-millimeter corrections to the Newton's law. We comment on the early universe cosmology of such models.

Keywords: Field Theories in Higher Dimensions, Extra Large Dimensions, Cosmology of Theories beyond the SM, Compactification and String Models.

Contents

1. Introduction

Theories with large extra dimensions provide a new framework for addressing the gauge hierarchy problem [1]. There are examples how such frameworks may arise from string theory compactifications [2, 3]. Much interest has been devoted to the models with an exponentially warped extra dimension [4], where the hierarchy arises from the gravitational redshift due to the curvature in the bulk rather than from the sheer size of the extra dimensions. These models can be linked with AdS/CFT correspondence in string theory [5], especially in the case when the extra dimension is of infinite proper size, but ends on a brane in the UV [6]. In this case, one encounters a new mechanism for generating 4D gravity out of infinite, noncompact extra dimensions. By respecting boundary conditions on the UV brane consistent with 4D general covariance, one finds quite generally that there is a normalizable gravitational mode localized on the UV brane, whose exchange generates 4D gravitational force [6, 7]. The localized graviton mode persists for a large class of intrinsic geometries on the UV brane, most notably for de Sitter brane [8], lending to the construction of interesting cosmologies. This phenomenon of gravity localization does not depend on the codimension of the UV brane. Four-dimensional gravity can also be localized on intersections of codimension-one branes on a higher-codimension brane in an $AdS_{4+n}, n > 1$ environment [9]. Some simple extensions of the example [9] were considered in [10]. However, one needs to consider the general case where all the branes have nonzero tension in order to address issues of possible vacua, stability, cosmological evolution and general multi-brane setups [8], [11]–[17].

In this paper we explicitly derive a general class of solutions describing 4-branes intersecting on a tensionful 3-brane in a locally AdS_6 environment, such that the intrinsic 3-brane geometry is maximally symmetric, being Minkowski, de Sitter or Anti-de Sitter. These exhaust the vacua on the 3-brane. In the case of Minkowski and de Sitter they localize 4D gravity to the intersection. The solutions resemble an infinitely tall 4-sided pyramid. We find them by cutting and pasting sections of AdS_6 bulk, such that the 4-branes reside at the seams and the 3-brane at the tip. The bulk cosmological constant is really a constant, rather than a step potential, since the AdS patches between the branes are locally identical, and hence we can orbifold the configuration by its discrete symmetries. Our solutions can be straightforwardly generalized to the case of AdS_{4+n}, $n > 2$, where n $n+2$-branes intersect on a 3-brane, with a more general relation between brane tensions and angles between them. For the solutions with Minkowski metric along the intersection of 4-branes with identical tensions, we find the 4D graviton zero mode localized to the intersection, explicitly solving for its wavefunction, and we compute the couplings of the states in the Kaluza-Klein continuum to the matter stress-energy on the intersection. Just like in the Randall-Sundrum case [6, 18], the couplings of the continuum modes are suppressed due to the warping of the bulk, except in this case as $\sim m^2L^2$, yielding them negligible at long range. Indeed, at large distances, $r \gg L$ the leading order correction to the Newton's law is softened by additional powers of L/r because of the tunnelling suppression, $\delta V \sim -G_N \frac{m_1 m_2 L^5}{r^6}$. This shows explicitly that at large distances the objects localized to the 3-brane interact with the usual 4D newtonian gravitational force. Similar situation persists for de Sitter origami, because the bulk volume is finite. We comment on the cosmic history of these models, indicating how the usual 4D FRW universes could be recovered.

2. Brane origamido

Imagine an array of two 4-branes with tensions σ_1 and σ_2 intersecting on a 3-brane with tension λ, all of them positive, in a locally AdS_6 bulk with a negative, really constant, cosmological term Λ (unlike in some of the ref. [10]). We will construct the solution describing this array of branes by patching together identical pieces of AdS_6 space, placing the branes on the seams of the bulk patchwork. Because the branes are infinitely thin, they are merely setting the boundary conditions for the bulk, which is locally the same AdS_6 anywhere away from the branes. The brane equations of motion are automatically solved once the covariant boundary conditions are enforced. We can orbifold the configuration by using the discrete symmetries of the structure, which are in general the rotations along the intersection and reflections around the 4-branes. Once we patch together the bulk from the AdS_6 fragments in a way consistent with all the symmetries, we can immediately read off the metric. To relate its geometric properties to the tensions of the branes on the seams, we use the field equations, which can be derived from the action

$$S = \int_M d^6x\sqrt{g_6}\left(\frac{1}{2\kappa_6^2}R + \Lambda\right) - \sum_{k=1}^{2}\sigma_k \int d^5x\sqrt{g_5^{(k)}} - \lambda\int d^4x\sqrt{g_4} + \text{boundary terms}\,. \quad (2.1)$$

The boundary terms are a generalization of the familiar Gibbons-Hawking terms needed to properly covariantize the action on manifolds with singular boundaries. Such terms have been discussed in [19]. Here $\kappa_6^2 = 1/{M_*}^4$, where M_* is the fundamental scale of the theory. The measure of integration in individual contributions to the action differs between each brane, and between the branes and the bulk, reflecting different codimensions of the sources. The terms $g_4, g_5^{(k)}$ are the determinants of the induced metrics on the 3- and 4-branes, respectively. In the field equations, this yields the ratios $\sqrt{g_4/g_6}$, $\sqrt{g_5^{(k)}/g_6}$ which weigh the δ-function sources. The field equations are formally

$$\begin{aligned} R^A{}_B - \frac{1}{2}\delta^A{}_B R = \kappa_6^2 \Lambda \delta^A{}_B &- \frac{\sqrt{g_4}}{\sqrt{g_6}}\kappa_6^2\, \lambda\, \delta(z_1)\delta(z_2)\,\mathrm{diag}(1,1,1,1,0,0) - \\ &- \frac{\sqrt{g_5^{(1)}}}{\sqrt{g_6}}\kappa_6^2\, \sigma_1\, \delta(z_1)\,\mathrm{diag}(1,1,1,1,0,1) - \\ &- \frac{\sqrt{g_5^{(2)}}}{\sqrt{g_6}}\kappa_6^2\, \sigma_2\, \delta(z_2)\,\mathrm{diag}(1,1,1,1,1,0)\,, \end{aligned} \tag{2.2}$$

where the coordinates z_1, z_2 parameterize the dimensions along the two 4-branes, that need not be orthogonal, and indices A, B run over all 6D. The sources take the particularly simple form above because we choose to write the Einstein's equations in the mixed tensor form, where the metric tensors are always equal to unity. The brane equations of motion are accounted for in (2.2) via the Bianchi identities. Tracing this out and eliminating the Ricci scalar, the Ricci tensor becomes

$$\begin{aligned} R^A{}_B = -\frac{\kappa_6^2\Lambda}{2}\delta^A{}_B &+ \frac{\sqrt{g_4}}{\sqrt{g_6}}\kappa_6^2\, \lambda\, \delta(z_1)\delta(z_2)\,\mathrm{diag}(0,0,0,0,1,1) + \\ &+ \frac{\sqrt{g_5^{(1)}}}{\sqrt{g_6}}\frac{\kappa_6^2\,\sigma_1}{4}\, \delta(z_1)\,\mathrm{diag}(1,1,1,1,5,1) + \\ &+ \frac{\sqrt{g_5^{(2)}}}{\sqrt{g_6}}\frac{\kappa_6^2\,\sigma_2}{4}\, \delta(z_2)\,\mathrm{diag}(1,1,1,1,1,5)\,. \end{aligned} \tag{2.3}$$

Upon completing the patching of the AdS_6 with the branes, we will substitute the solution for the metric into (2.3), and simply read off the relations between the tensions and the folding angles.

Let us now turn to bulk surgery. We start with the construction of the Minkowski intersection, embedded in AdS_6 with the metric in the Poincare coordinates:

$$ds_6^2 = \frac{L^2}{w_1^2}\left(\eta_{\mu\nu}dx^\mu dx^\nu + dw_2^2 + dw_1^2\right). \tag{2.4}$$

Here w_1 is the radial coordinate in AdS_6, with the AdS boundary at $w_1 = 0$ and the AdS 'infinity' at $w_1 \to \infty$, and w_2 is a spatial coordinate parallel with the boundary. The Greek indices μ, ν denote the 4D coordinates along the intersection. It is convenient to parameterize the $\{w_1, w_2\}$ plane by vectors $\vec{w} = (w_1, w_2)$ and introduce the vector $\vec{n} = (1, 0)$, such that $w_1 = \vec{n}\cdot\vec{w}$. Then the metric (2.4) is $ds_6^2 = [L^2/(\vec{n}\cdot\vec{w})^2](\eta_{\mu\nu}dx^\mu dx^\nu + d\vec{w}^2)$. Now take two non-coincident 4-branes and place them in the AdS bulk at angles to $\vec{n}$ which differ from zero, such that they straddle the radial axis $w_2 = 0$. Let them intersect at a point $(w_{1\,0}, 0)$ on the radial axis. We can always shift the intersection to $w_{1\,0} = L$ [6], accompanying this by a rescaling of the brane-localized lagrangians. Changing the

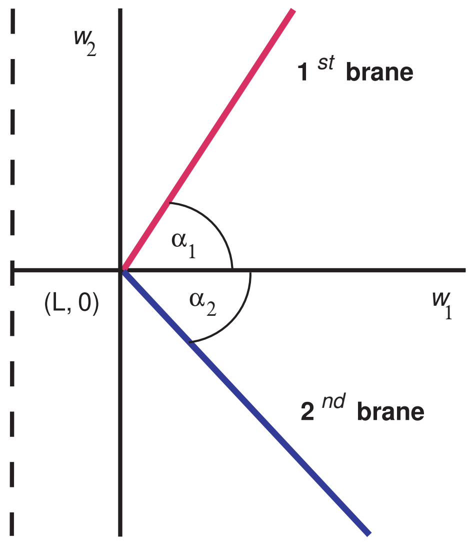

Figure 1: Two 4-branes intersecting in a Poincare patch of AdS_6.

coordinates according to $\vec{n} \cdot \vec{w} \to \vec{n} \cdot \vec{w}' + L$ and dropping the primes we get

$$ds_6^2 = \frac{L^2}{(\vec{n} \cdot \vec{w} + L)^2} \left(\eta_{\mu\nu} dx^\mu dx^\nu + d\vec{w}^2\right) . \tag{2.5}$$

In this coordinate system, the unit vectors $\vec{e}_k$, $k \in \{1, 2\}$, pointing along the two 4-branes (see figure 1) are given by $\vec{e}_1 = (\cos\alpha_1, \sin\alpha_1)$ and $\vec{e}_2 = (\cos\alpha_2, -\sin\alpha_2)$, where α_1 and α_2 are the absolute values of the angles between the 4-branes and the radial axis $w_2 = 0$. The unit normals to the two 4-branes $\vec{n}_k$, pointed towards AdS infinity, are defined as points on a unit circle,

$$\vec{n}_1 = (\sin\alpha_1, -\cos\alpha_1) , \qquad \vec{n}_2 = (\sin\alpha_2, \cos\alpha_2) . \tag{2.6}$$

We can now define the duals of the basis $\{\vec{n}_k\}$, denoted $\{\vec{l}_k\}$, by the relation

$$\vec{l}_k \cdot \vec{n}_l = \delta_{kl} . \tag{2.7}$$

Using (2.6), we find

$$\vec{l}_1 = \frac{1}{\sin(\alpha_1 + \alpha_2)} (\cos\alpha_2, -\sin\alpha_2) , \qquad \vec{l}_2 = \frac{1}{\sin(\alpha_1 + \alpha_2)} (\cos\alpha_1, \sin\alpha_1) , \tag{2.8}$$

or $\vec{l}_1 = \vec{e}_2 / \sin(\alpha_1 + \alpha_2)$ and $\vec{l}_2 = \vec{e}_1 / \sin(\alpha_1 + \alpha_2)$. Because $\vec{n}_k$ are not orthonormal, $\vec{l}_k$ are not unit vectors, except when $\alpha_1 + \alpha_2 = \pi/2$. Because $\{\vec{n}_k\}$ and $\{\vec{l}_k\}$ are duals, we have the completeness relation

$$\sum_{k=1}^{2} (\vec{l}_k)_i (\vec{n}_k)_j = \delta_{ij} , \tag{2.9}$$

where i denotes i-th component of the vector $\vec{l}_k$ etc.

The branes are localized at hypersurfaces $\vec{n}_k \cdot \vec{w} = 0$. We now use $\vec{n}_k$ as the basis of the 2D space between the two 4-branes, and define new coordinates

$$\tilde{z}_k = \vec{n}_k \cdot \vec{w} , \qquad k \in \{1, 2\} . \tag{2.10}$$

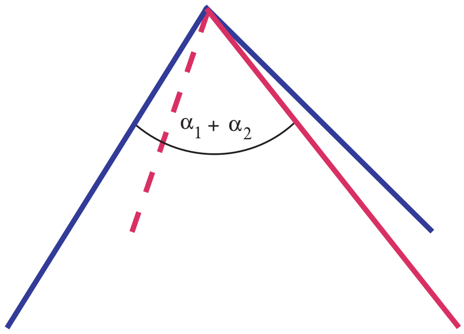

Figure 2: Brane pyramid composed of AdS_6 patches.

In this basis $\vec{n} = \sum_{k=1}^{2} \mathcal{C}_k \vec{n}_k$, where $\mathcal{C}_k = \vec{l}_k \cdot \vec{n}$. Therefore $\mathcal{C}_1 = \cos\alpha_2 / \sin(\alpha_1 + \alpha_2)$ and $\mathcal{C}_2 = \cos\alpha_1 / \sin(\alpha_1 + \alpha_2)$, and hence $\vec{n} \cdot \vec{w} = \sum_{k=1}^{2} \mathcal{C}_k \vec{n}_k \cdot \vec{w} = \mathcal{C}_1 \tilde{z}_1 + \mathcal{C}_2 \tilde{z}_2$. Using the completeness relation (2.9), we invert (2.10): $w_j = \sum_{k=1}^{2} \tilde{z}_k (\vec{l}_k)_j$, $j \in \{1, 2\}$, or

$$\vec{w} = \sum_{k=1}^{2} \tilde{z}_k \vec{l}_k \,. \tag{2.11}$$

From (2.11) it is clear[1] that the coordinates $\tilde{z}_k$ measure the distance from one 4-brane along the other. Since $d\vec{w}^2 = \sum_{k,l} \vec{l}_k \cdot \vec{l}_l \, d\tilde{z}_k d\tilde{z}_l$, we can rewrite the metric (2.5) as

$$ds_6^2 = \frac{L^2}{(\sum_{k=1}^{2} \mathcal{C}_k \tilde{z}_k + L)^2} \left(\eta_{\mu\nu} dx^\mu dx^\nu + \sum_{k,l=1}^{2} \vec{l}_k \cdot \vec{l}_l \, d\tilde{z}_k d\tilde{z}_l \right) . \tag{2.12}$$

The metric (2.12) covers the region both between the branes and outside of them, on the side of the AdS boundary. We need to cut out the region between the 4-branes and the AdS boundary out in order to have a normalizable 4D graviton, localized to the intersection, because this region has infinite proper volume. To do this we take the slice bounded by the branes and reflect it around the branes to build the bulk region which looks like a pyramid with branes at the edges (see figure 2). This corresponds to retaining only the patch of (2.12) covered by $\tilde{z}_k \geq 0$, and flipping the direction of the coordinate axis every time a 4-brane is crossed, while keeping the values of the coordinate units fixed. The metric of the folded structure, or brane origami, can be found by substituting in (2.12) the new coordinates

$$\tilde{z}_k \to \tilde{z}_k = |z_k| \,, \tag{2.13}$$

which is not a diffeomorphism, but a coordinate restriction. Since $d|z_k| = \mathtt{sgn}(z_k) dz_k + 2\delta(z_k) z_k dz_k = \mathtt{sgn}(z_k) dz_k$, where $\mathtt{sgn}(x) = 2\theta(x) - 1$ is the sign function, and $\theta(x)$ is the step function, the metric of the intersection of two arbitrary 4-branes on a tensionful

[1] Our choice of coordinate labels "1" and "2" as being along $\vec{l}_1, \vec{l}_2$, respectively, implies that the "1st" 4-brane is orthogonal to the normal $\vec{n}_1$ and the "2nd" 4-brane is orthogonal to $\vec{n}_2$ (see figure 1). Thus for notational reasons the parameter $\mathcal{C}_2$ is related to the orientation of the 1st 4-brane (i.e. the angle α_1, as we see above), and $\mathcal{C}_1$ is related to the orientation of the 2nd 4-brane. But because the tension of a codimension-one brane measures the normal gradient of the bulk metric on the brane, according to the junction conditions, the tension of the 1st 4-brane is mostly determined by $\mathcal{C}_1$, and the tension of the 2nd by $\mathcal{C}_2$, as we will show later.

3-brane is

$$ds_6^2 = \frac{L^2}{(\sum_{k=1}^2 \mathcal{C}_k |z_k| + L)^2} \left(\eta_{\mu\nu} dx^\mu dx^\nu + \sum_{k,l=1}^2 \bar{g}_{kl}(z_n)\, dz_k dz_l \right), \tag{2.14}$$

where the 2D transverse metric $\bar{g}_{kl}(z_n)$ is given by the matrix

$$(\bar{g}_{kl})\,(z_n) = \begin{pmatrix} \vec{l}_1^{\,2} & \vec{l}_1 \cdot \vec{l}_2 \, \mathtt{sgn}(z_1)\mathtt{sgn}(z_2) \\ \vec{l}_1 \cdot \vec{l}_2 \, \mathtt{sgn}(z_1)\mathtt{sgn}(z_2) & \vec{l}_2^{\,2} \end{pmatrix}. \tag{2.15}$$

We stress that because of the construction of (2.14) by the reflections *around* the 4-branes, each time a brane is crossed we only flip the sign of the cross term in (2.15) and do not change the diagonal terms.

In general, the origami (2.14) is composed of four distinct but locally identical AdS_6 patches. We can now orbifold it by identifying the points which are related by discrete symmetries of (2.14). The discrete symmetries available are the reflections $z_k \leftrightarrow -z_k$, and the two rotations around the intersection by the angle $\alpha_1 + \alpha_2$. In the special case of identical 4-branes, the angles α_1 and α_2 are the same, and so the set of rotations can be enlarged to encompass the four rotations by 2α forming Z_4, of cyclic permutations on 4 elements. We will consider orbifolding in detail later on, when we turn to the spectrum of bulk gravitons in the background (2.14).

The solutions with de Sitter geometry intrinsic to the 3-brane are interesting for cosmological model-building, generalizing the bent domain walls of [8]. Finding them is a straightforward extension of the folding procedure outlined above. Since the metric (2.4) is conformally flat, a boost in the t, w_1 plane,

$$t \to t' = \mathcal{C}t - \mathcal{S}w_1\,, \qquad w_1 \to w_1' = \mathcal{C}w_1 - \mathcal{S}t\,, \tag{2.16}$$

where $\mathcal{C} = \cosh\gamma$ and $\mathcal{S} = \sinh\gamma$, only changes the conformal factor. After it, having rewritten the metric in terms of w', t' and having dropped the primes, we find $ds_6^2 = [L^2/(\mathcal{C}w_1 + \mathcal{S}t)^2](\eta_{\mu\nu}dx^\mu dx^\nu + dw_2^2 + dw_1^2)$. Starting with this metric, the rest of the procedure is then part-way identical to our previous construction, up to eq. (2.12). Introducing the same coordinates $\tilde{z}_k$ as before, we arrive at

$$ds_6^2 = \frac{L^2}{(\mathcal{C}\sum_{k=1}^2 \mathcal{C}_k \tilde{z}_k + \mathcal{S}t + L)^2} \left(\eta_{\mu\nu} dx^\mu dx^\nu + \sum_{k,l=1}^2 \vec{l}_k \cdot \vec{l}_l \, d\tilde{z}_k d\tilde{z}_l \right). \tag{2.17}$$

We can remove L in the denominator of the conformal factor by a time translation $t \to t - L/\mathcal{S}$. In the coordinate system that yields the metric (2.17), the intersection moves through the bulk with a constant radial speed $\dot{w} = \mathcal{S}/\mathcal{C}$ relative to the manifestly static case in (2.14), while the metric intrinsic to the intersection appears flat. However, a radial translation in AdS corresponds to a mass rescaling along the branes [6], which is one of the cornerstones of the AdS/CFT correspondence [5]. If we denote the conformal factor (i.e. warp factor) in (2.17) by Ω, the masses of probes along the intersection $\tilde{z}_k = 0$ change in time according to $m(t) = m_0\Omega(\mathcal{S}t)$. Thus the unit length, defined by the

Compton wavelength of a reference particle, e.g a proton, changes according to $\lambda(t) = \lambda_0/\Omega(\mathcal{S}t)$. However, *all* the particle mass scales along the intersection transform in exactly the same way, and so their ratios remain constant. Thus the apparent time dependence of the unit length is a coordinate effect, completely analogous to what one would find in the conventional 4D cosmology were she to absorb the cosmological scale factor into the definition of particle masses. While possible, this is neither elegant nor convenient. It is much better to coordinatize the geometry so that the relevant units are time-independent. This can be readily accomplished by the coordinate map

$$\begin{aligned}
\tilde{z}_k &= \frac{\mathcal{S}e^{-\mathcal{S}T/L}}{\sqrt{1-\frac{\mathcal{S}^2}{L^2}\sum_{k,l=1}^{2}\vec{l}_k\cdot\vec{l}_l\,\hat{z}_l\hat{z}_k}}\hat{z}_k\,, \\
t &= \frac{L}{\sqrt{1-\frac{\mathcal{S}^2}{L^2}\sum_{k,l=1}^{2}\vec{l}_k\cdot\vec{l}_l\,\hat{z}_l\hat{z}_k}}e^{-\mathcal{S}T/L}-\frac{L}{\mathcal{S}}\,, \\
\vec{x} &= \mathcal{S}\vec{X}\,.
\end{aligned} \tag{2.18}$$

In terms of the new coordinates $\hat{z}_k, T$ and $\vec{X}$, the metric becomes

$$\begin{aligned}
ds_6^2 = \frac{L^2}{(\mathcal{C}\sum_{k=1}^2 \mathcal{C}_k\hat{z}_k+L)^2}\Big\{&\Big(1-\frac{\mathcal{S}^2}{L^2}\sum_{k,l=1}^{2}\vec{l}_k\cdot\vec{l}_l\,\hat{z}_k\hat{z}_l\Big)\Big[-dT^2+e^{2\mathcal{S}T/L}d\vec{X}^2\Big]+ \\
&+\sum_{k,l=1}^{2}\Big[\vec{l}_k\cdot\vec{l}_l+\frac{\mathcal{S}^2}{L^2}\frac{\sum_{m,n=1}^{2}\hat{z}_m\hat{z}_n\,\vec{l}_k\cdot\vec{l}_m\;\vec{l}_l\cdot\vec{l}_n}{1-\frac{\mathcal{S}^2}{L^2}\sum_{m,n=1}^{2}\vec{l}_m\cdot\vec{l}_n\,\hat{z}_m\hat{z}_n}\Big]d\hat{z}_kd\hat{z}_l\Big\}\,.
\end{aligned} \tag{2.19}$$

Cutting and folding can now be done precisely in the same way as in the static case, by keeping only the region covered by $\hat{z}_k \ge 0$, by a map like (2.13). We take $\hat{z}_k \to \hat{z}_k = |z_k|$, $d\hat{z}_k \to d\hat{z}_k = \mathtt{sgn}(z_k)dz_k$, substitute it in (2.19) and find the metric of de Sitter origami

$$\begin{aligned}
ds_6^2 = &\frac{L^2}{(\mathcal{C}\sum_{k=1}^2 \mathcal{C}_k|z_k|+L)^2}\times \\
&\times\Big\{\Big(1-\frac{\mathcal{S}^2}{L^2}\sum_{k,l=1}^{2}\vec{l}_k\cdot\vec{l}_l\,|z_k||z_l|\Big)\Big[-dT^2+e^{2\mathcal{S}T/L}d\vec{X}^2\Big]+ \\
&+\sum_{k,l=1}^{2}\Big[\bar{g}_{kl}(z_n)+\frac{\mathcal{S}^2}{L^2}\frac{\sum_{m,n=1}^{2}|z_m||z_n|\,\vec{l}_k\cdot\vec{l}_m\;\vec{l}_l\cdot\vec{l}_n\mathtt{sgn}(z_k)\mathtt{sgn}(z_l)}{1-\frac{\mathcal{S}^2}{L^2}\sum_{m,n=1}^{2}\vec{l}_m\cdot\vec{l}_n\,|z_m||z_n|}\Big]dz_kdz_l\Big\}\,,
\end{aligned} \tag{2.20}$$

where $\bar{g}_{kl}(z_n)$ is given in eq. (2.15). The metric intrinsic to the intersection of 4-branes at $z_k = 0$ in (2.20) is indeed de Sitter, with a Hubble scale $H = \mathcal{S}/L$. Here we have used the spatially flat slicing for simplicity, but one can easily go to other coordinate coverings of de Sitter. The warping is now time-independent, and so the units along the intersection are also constant. We can orbifold this structure in a way analogous to the Minkowski origami (2.14). In the limit $\mathcal{S} \to 0$, $\mathcal{C} \to 1$, the de Sitter origami (2.20) smoothly deforms into the Minkowski origami (2.14). Unlike in (2.14), where the bulk Poincare patch Cauchy horizon, given by the limit $\sum_{k=1}^2 \mathcal{C}_k z_k \to \infty$, resides at infinite proper distance from the intersection, in the de Sitter origami case it is located at $\sum_{k,l=1}^{2} \vec{l}_k \cdot \vec{l}_l \, |z_k||z_l| = L^2/\mathcal{S}^2$, a finite proper distance $\sim L/\mathcal{S}$ from the intersection. In the limit $\mathcal{S} \to 0$, $\mathcal{C} \to 1$ the horizon moves to infinity, as in the case of a single Minkowski and de Sitter brane in AdS_5 [6, 8].

The Anti-de Sitter origami, with AdS_4 spacetime along the 3-brane, can be constructed in a way very similar to the de Sitter origami. Starting with the AdS_6 metric (2.4), instead of a boost in the t, w_1 plane (2.16), perform a rotation in the plane defined by one of the spatial coordinates along the intersection, say x^3, and w_1. Then repeat the steps which led to (2.20). The result can actually be found faster, by taking (2.20) and performing a double Wick rotation $T \to iX^3$, $X^3 \to -iT$, while simultaneously taking the boost angle γ in (2.16) to be imaginary, $\gamma = i\bar{\gamma}$. Defining $\bar{\mathcal{C}} = \cos\bar{\gamma}$ and $\bar{\mathcal{S}} = \sin\bar{\gamma}$, and using $\bar{g}_{kl}$ in (2.15), the AdS_4 origami is

$$\begin{aligned} ds_6^2 = & \frac{L^2}{(\bar{\mathcal{C}} \sum_{k=1}^{2} \mathcal{C}_k |z_k| + L)^2} \times \\ & \times \left\{ \left(1 + \frac{\bar{\mathcal{S}}^2}{L^2} \sum_{k,l=1}^{2} \vec{l}_k \cdot \vec{l}_l \, |z_k||z_l| \right) \left[d(X^3)^2 + e^{-2\bar{\mathcal{S}} X^3/L} \left(\sum_{k=1}^{2} d(X^k)^2 - dT^2 \right) \right] + \right. \\ & \left. + \sum_{k,l=1}^{2} \left[\bar{g}_{kl}(z_n) - \frac{\bar{\mathcal{S}}^2}{L^2} \frac{\sum_{m,n=1}^{2} |z_m||z_n| \, \vec{l}_k \cdot \vec{l}_m \; \vec{l}_l \cdot \vec{l}_n \, \mathrm{sgn}(z_k) \mathrm{sgn}(z_l)}{1 + \frac{\bar{\mathcal{S}}^2}{L^2} \sum_{m,n=1}^{2} \vec{l}_m \cdot \vec{l}_n \, |z_m||z_n|} \right] dz_k dz_l \right\} . \end{aligned} \tag{2.21}$$

The important property of this solution is that there is no horizon in the bulk surrounding the intersection, and hence far from the branes the bulk geometry opens up and encloses an infinite portion of the bulk volume near the AdS_6 boundary, in the limit $z_k \to \infty$. Therefore the solution (2.21) does not localize 4D gravity at the intersection. Because of this, we will focus on the Minkowski and de Sitter origami in what follows. However it would be interesting to determine if it leads to the phenomenon of quasilocalization, found in the case of AdS_4 brane in AdS_5 [20].

Now we prove that the origami configuration (2.14), (2.20) does solve the field equations (2.3) describing the intersections of 4-branes on a tensionful 3-brane. First, note that the de Sitter origami can be described either by (2.20) or by (2.17) after imposing the restriction $\tilde{z}_k \to \tilde{z}_k = |z'_k|$. The metric is, using $\bar{g}_{kl}(z'_n)$ from (2.15), with z'_k in place of z_k,

$$ds_6^2 = \frac{L^2}{(\mathcal{C} \sum_{k=1}^{2} \mathcal{C}_k |z'_k| + \mathcal{S} t + L)^2} \left(\eta_{\mu\nu} dx^\mu dx^\nu + \sum_{k,l=1}^{2} \bar{g}_{kl}(z'_n) \, dz'_k dz'_l \right) . \tag{2.22}$$

This equivalence follows from the coordinate map (2.18), which shows that the spaces metricized by (2.20) and (2.22) are in one-to-one correspondence, because the coordinates z'_k and z_k are related in the same way as $\tilde{z}_k$ and $\hat{z}_k$. This metric reduces to (2.14) in the limit $\mathcal{S} \to 0$, $\mathcal{C} \to 1$. While the metric (2.22) is not as transparent for describing the physics of these solutions, as we have explained above, it is better suited for proving that it solves the field equations (2.3). Once this is established, the diffeomorphism (2.18) guarantees that (2.20) is also a solution. To show this, we can evaluate explicitly the Ricci tensor of (2.22) in two simple steps. First, notice that (2.22) is conformal to a metric which is flat almost everywhere except at the location of the branes: $ds_6^2 = \Omega^2 d\bar{s}_6^2$ where $d\bar{s}_6^2 = \eta_{\mu\nu}dx^\mu dx^\nu + \sum_{k,l=1}^2 \bar{g}_{kl}(z'_n)\, dz'_k dz'_l$. Therefore its Ricci tensor is given by

$$R_{AB} = \bar{R}_{AB} - 4\bar{\nabla}_A\bar{\nabla}_B(\ln\Omega) - \bar{g}_{AB}\bar{\nabla}^2(\ln\Omega) + 4\bar{\nabla}_A(\ln\Omega)\bar{\nabla}_B(\ln\Omega) - 4\bar{g}_{AB}(\bar{\nabla}\ln\Omega)^2\,, \quad (2.23)$$

where the barred quantities refer to the metric $d\bar{s}_6^2$. Next, note that $d\bar{s}_6^2$ splits in a direct sum of 4D Minkowski metric and the metric on the 2D Euclidean wedge $z'_k > 0$, depending only on the 2D coordinates. Thus $\bar{R}_{\mu\nu} = \bar{R}_{k\mu} = 0$. The 2D part is fully determined by the 2D Ricci scalar because the Einstein tensor is identically zero in two dimensions: $\bar{R}_{kl} = \frac{1}{2}\bar{g}_{kl}\bar{R}$. Thus to fully determine $\bar{R}_{kl}$ it is sufficient to compute only one of its components. Because the 2D transverse metric in (2.15) is defined in the distributional sense, with sign functions whose derivatives are δ-functions, this computation requires a little care. From the definitions of the dual bases $\{\vec{n}_k\}$ and $\{\vec{l}_k\}$, and because $\vec{n}_k^2 = 1$ by definition (4.6), the inverse 2D metric is defined according to

$$\left(\bar{g}^{kl}\right)(z'_n) = \begin{pmatrix} 1 & \vec{n}_1\cdot\vec{n}_2\,\mathtt{sgn}(z'_1)\mathtt{sgn}(z'_2) \\ \vec{n}_1\cdot\vec{n}_2\,\mathtt{sgn}(z'_1)\mathtt{sgn}(z'_2) & 1 \end{pmatrix}. \quad (2.24)$$

Because (2.15) and (2.24) are inverses, the entries in (2.15) are $\vec{l}_l^2 = 1/[1-(\vec{n}_1\cdot\vec{n}_2)^2]$ and $\vec{l}_1\cdot\vec{l}_2 = -\vec{n}_1\cdot\vec{n}_2/[1-(\vec{n}_1\cdot\vec{n}_2)^2]$. Finally the step and sign functions obey distributional rules $\frac{d\mathtt{sgn}(x)}{dx} = 2\delta(x)$, $[\mathtt{sgn}(x)]^2 = 1$ and $\mathtt{sgn}(x)\delta(x) = 0$. Using this, the Christoffel symbols which do not vanish identically are

$$\bar{\Gamma}^k_{kk} = -2\frac{(\vec{n}_1\cdot\vec{n}_2)^2}{1-(\vec{n}_1\cdot\vec{n}_2)^2}\mathtt{sgn}(z'_k)\delta(z'_k)\,, \qquad \bar{\Gamma}^l_{kk} = -2\frac{\vec{n}_1\cdot\vec{n}_2}{1-(\vec{n}_1\cdot\vec{n}_2)^2}\mathtt{sgn}(z'_l)\delta(z'_k)\,. \quad (2.25)$$

We keep the terms $\hat{\Gamma}^k_{kk} \propto \mathtt{sgn}(z'_k)\delta(z'_k)$, because while they vanish when left to their own, they may contribute to quantities such as curvature when multiplied by terms $\propto \mathtt{sgn}(z'_k)$. This yields, for example $\bar{R}_{11} = \partial_2\bar{\Gamma}^2_{11} + \bar{\Gamma}^2_{22}\bar{\Gamma}^2_{11} = -4\frac{\vec{n}_1\cdot\vec{n}_2}{[1-(\vec{n}_1\cdot\vec{n}_2)^2]^2}\,\delta(z'_1)\delta(z'_2) = \frac{1}{2}\bar{g}_{11}\bar{R}$, and so the 2D Ricci curvature is

$$\bar{R} = -8\,\frac{\vec{n}_1\cdot\vec{n}_2}{1-(\vec{n}_1\cdot\vec{n}_2)^2}\,\delta(z'_1)\,\delta(z'_2)\,, \quad (2.26)$$

i.e. exactly of the form $\propto \delta(z'_1)\delta(z'_2)$ at the origin, as required to describe a 3-brane with nonzero tension located on the intersection. The last step is to conformally transform the Ricci tensor, using $g_{AB} = \Omega^2\bar{g}_{AB}$ where $\Omega = L/(\mathcal{C}\sum_{k=1}^2 \mathcal{C}_k|z'_k| + \mathcal{S}t + L)$, and where the Ricci tensors are related according to (2.23). A straightforward albeit tedious calculation[2] yields, for the mixed-index Ricci tensor, the following expression:

$$\begin{aligned} R^A{}_B = &-\frac{5}{L^2}\delta^A{}_B - \frac{4}{\Omega^2|_{z'_k=0}}\,\frac{\vec{n}_1\cdot\vec{n}_2}{1-(\vec{n}_1\cdot\vec{n}_2)^2}\,\delta(z'_1)\delta(z'_2)\,\mathrm{diag}(0,0,0,0,1,1) + \\ &+\frac{2\mathcal{C}}{L\Omega|_{z'_1=0}}\,\frac{1}{1-(\vec{n}_1\cdot\vec{n}_2)^2}\Big(\mathcal{C}_1 + \mathcal{C}_2\,\vec{n}_1\cdot\vec{n}_2\Big)\,\delta(z'_1)\,\mathrm{diag}(1,1,1,1,5,1) + \\ &+\frac{2\mathcal{C}}{L\Omega|_{z'_2=0}}\,\frac{1}{1-(\vec{n}_1\cdot\vec{n}_2)^2}\Big(\mathcal{C}_2 + \mathcal{C}_1\,\vec{n}_1\cdot\vec{n}_2\Big)\,\delta(z'_2)\,\mathrm{diag}(1,1,1,1,1,5)\,. \end{aligned} \quad (2.27)$$

[2] By definition of $\vec{n} = \sum_{k=1}^2 \mathcal{C}_k\vec{n}_k = (1,0)$ we have $\vec{n}^2 = \sum_{k,l=1}^2 \mathcal{C}_k\mathcal{C}_l\vec{n}_k\cdot\vec{n}_l = 1$. This comes in handy when computing the covariant derivatives of Ω.

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The AdS radius L and the bulk cosmological constant Λ are related by the usual formula, which in 6D is $\kappa_6^2\Lambda = 10/L^2$. Taking into account the ratios of volume factors $\sqrt{g_4/g_6} = \sqrt{1-(\vec{n}_1\cdot\vec{n}_2)^2}/\Omega^2$ and $\sqrt{g_5^{(1)}/g_6} = \sqrt{g_5^{(2)}/g_6} = 1/\Omega$ and dropping the irrelevant primes from z'_k in (2.27), we see that the equation (2.27) is identical with (2.3) if the brane tensions and the geometric structure parameters satisfy

$$\begin{aligned}
\kappa_6^2\lambda &= -4\,\frac{\vec{n}_1\cdot\vec{n}_2}{(1-(\vec{n}_1\cdot\vec{n}_2)^2)^{3/2}}\,,\\
\kappa_6^2\sigma_1 &= 8\,\frac{\mathcal{C}}{L}\,\frac{1}{1-(\vec{n}_1\cdot\vec{n}_2)^2}\,(\mathcal{C}_1+\mathcal{C}_2\,\vec{n}_1\cdot\vec{n}_2)\,,\\
\kappa_6^2\sigma_2 &= 8\,\frac{\mathcal{C}}{L}\,\frac{1}{1-(\vec{n}_1\cdot\vec{n}_2)^2}\,(\mathcal{C}_2+\mathcal{C}_1\,\vec{n}_1\cdot\vec{n}_2)\,.
\end{aligned}\tag{2.28}$$

This completes our proof that the de Sitter origami family solves the field equations (2.2). This family reduces to the Minkowski origami (2.14), which is therefore also covered by our proof. Finally, a similar calculation shows that the AdS origami (2.21) also solves the field equations (2.2), once the appropriate Wick rotations of parameters are substituted.

As a corollary, we have determined the correspondence between the tensions of the branes and the angles between them, as given in (2.28). Using the explicit formulas for $\vec{n}_k$ (2.6), $\vec{l}_k$ (2.8), $\mathcal{C}_k$ (in the text just below eq. (2.10)) and $\mathcal{C}$ (in the text just below eq. (2.16)), we can rewrite (2.28) explicitly in terms of angles α_1,α_2 and the boost parameter γ:

$$\begin{aligned}
\kappa_6^2\lambda &= 4\,\frac{\cos(\alpha_1+\alpha_2)}{\sin^3(\alpha_1+\alpha_2)}\,,\\
\kappa_6^2\sigma_1 &= 8\,\frac{\cosh\gamma}{L\sin^3(\alpha_1+\alpha_2)}\,\left(\cos\alpha_2-\cos\alpha_1\cos(\alpha_1+\alpha_2)\right)\,,\\
\kappa_6^2\sigma_2 &= 8\,\frac{\cosh\gamma}{L\sin^3(\alpha_1+\alpha_2)}\,\left(\cos\alpha_1-\cos\alpha_2\cos(\alpha_1+\alpha_2)\right)\,.
\end{aligned}\tag{2.29}$$

In order to satisfy the null energy conditions, which is a sufficient condition for the existence of a minimum energy state, one wants that all energy densities, including brane tensions, are non-negative. To ensure that the 3-brane tension is not negative, we should restrict $\cot(\alpha_1+\alpha_2)\geq 0$, i.e. $\alpha_k\leq\alpha_1+\alpha_2\leq\pi/2$. From (2.29) it then follows automatically that for if $\alpha_1,\alpha_2>0$, both 4-brane tensions are non-negative. However, if we had placed the 4-branes on the same side of the radial axis, say by moving the 2^{nd} brane above the radial axis (see figure 1), the direction of its normal would have had to be flipped relative to (4.6) by definition, since the normal should be pointed “outward”. This would have changed the overall sign in the last of (2.29), and so that brane would have had a negative tension. It would be interesting to carry out a more complete analysis of the general configurations with negative tensions to check explicitly for instabilities, however that is a task beyond the scope of the present work.

The procedure which we have employed to generate the solutions (2.14), (2.22) can be straightforwardly adopted to the case of any AdS_{4+n}, $n>2$, with n $n+2$-branes intersecting on a 3-brane of non-zero tension. Indeed, the only change in the formulas for

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the metric of the Minkowski origami (2.14) or the de Sitter origami (2.22) would be to change the range of summation over the coordinates transverse to the intersection from 2 to n. The form of the solutions would remain the same, as is clear from the implementation of the folding procedure. Instead of the pyramid structure in figure 2, one would get a higher-dimensional generalization, where the surfaces between the branes would be extended to higher-dimensional patches of AdS_{4+n}. The main difference would appear in the relationship of the angles between the branes and their tensions. The angles are defined by the normals on the 4-branes $\{\vec{n}_k\}$ and their duals $\{\vec{l}_k\}$, which in the case of AdS_{4+n} would be points on S^{n-1} rather than on a circle S^1.

3. Glimpses of cosmology

The equations (2.28), the definition of the AdS radius $L^2 = 10/[\kappa_6^2\Lambda]$ and the relation between the 4D Hubble scale and the boost parameter, $H = \sinh\gamma/L$, relate *five* physical scales λ, σ_1, σ_2, Λ and H to *four* integration constants $\gamma, \alpha_1, \alpha_2$ and L. Thus there must be one relation between them. If we express H as a function of the other scales, we find the effective 4D Hubble law, or Friedman equation. Simple algebra shows

$$H^2 = \frac{\kappa_6^4}{64}\left(1 - (\vec{n}_1\cdot\vec{n}_2)^2\right)\left\{\sigma_1^2 + \sigma_2^2 - 2\sigma_1\sigma_2\,\vec{n}_1\cdot\vec{n}_2\right\} - \frac{\kappa_6^2\Lambda}{10}\,, \tag{3.1}$$

where $\vec{n}_1\cdot\vec{n}_2$ is a negative square root of the nonnegative solution of the cubic equation

$$\frac{\kappa_6^4\lambda^2}{16}\Big(1 - (\vec{n}_1\cdot\vec{n}_2)^2\Big)^3 = (\vec{n}_1\cdot\vec{n}_2)^2\,. \tag{3.2}$$

These equations relate the 4D Hubble scale, or equivalently the effective 4D cosmological constant, to the tensions of the branes, and are analogous to the corresponding equation for the bent braneworlds in AdS_5 [8]. In the case of the Minkowski origami (2.14), where $H = 0$, this becomes the fine-tuning condition for the vanishing of the 4D cosmological constant, analogous to the one found in the RS2 case in AdS_5 [6], relating the brane tensions and the bulk cosmological constant:

$$\Lambda = \frac{5\kappa_6^2}{32}\left(1 - (\vec{n}_1\cdot\vec{n}_2)^2\right)\left\{\sigma_1^2 + \sigma_2^2 - 2\sigma_1\sigma_2\,\vec{n}_1\cdot\vec{n}_2\right\}. \tag{3.3}$$

The equation (3.1) hints at how the effective 4D cosmological evolution, governed by the standard 4D Einstein gravity at long distances, can emerge. In fact, there have been some attempts to recover 4D cosmology on codimension-2 braneworlds [21, 22], which were finding obstructions to the usual 4D cosmological evolution, and in some cases seeking for remedies by adding higher derivative operators in the bulk action. The origami setup which we have elaborated here can provide a natural and simple way around some of these difficulties and lead to a 4D cosmology at low energies. To outline how this should work, consider an approximately symmetric array of 4-branes, with $\sigma_1 \simeq \sigma_2$, and imagine that the brane tensions are hierarchically ordered, obeying[3] $\sigma_k L \gg \lambda \gg 0$. Further, pick the

[3] Note that it may be sufficient if these relations between tensions are realized within only a few orders of magnitude, e.g. that the tension of the 3-brane and the difference between the 4-brane tensions are of the order of a percent of their mean value. The systematic errors of our approximations would then be at most of the order of a percent as well, which should suffice to fit horizon-scale cosmology.

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tensions and the bulk cosmological constant such that they obey the fine tuning condition (3.3) so that the intrinsic 3-brane geometry is Minkowski, with $H = 0$. Then perturb the 3-brane at the intersection with a small amount of homogeneous brane-localized matter, of energy density ρ: $\lambda \to \lambda + \rho$. Thus the natural dimensionless expansion parameter is ρ/λ. From equations (2.29), this suggests that when the perturbation is turned on, the angles between the branes develop very slow time-dependence. To the leading order, from eq. (3.2), $\vec{n}_1 \cdot \vec{n}_2 \simeq -[\kappa_6^2\lambda/4](1 + \rho/\lambda)$. When $\sigma_k L \gg \lambda$, the contribution from the matter on 4-branes could be neglected, and so the perturbed geometry should look like an FRW universe with the Hubble parameter which, bearing in mind $\sigma_1 \simeq \sigma_2 \gg \lambda/L$ and using (2.29)–(3.2) is, to the leading order in ρ/λ,

$$H^2 = \frac{\kappa_6^4}{4L^2}\,\rho\,. \tag{3.4}$$

The approximations yielding (3.4) should get better with time, since the perturbation $\propto \rho$ redshifts away as the universe expands. The coefficient of ρ in (3.4) must be the inverse of $3{M_4}^2$ if this is to be the 4D Friedman equation; from this we obtain ${M_4}^2 = (4/3){M_*}^4 L^2$. Below we will see that this is indeed the correct answer in this limit, when we derive the 4D Planck constant from graviton perturbation theory. For general brane arrays, however, the crudeness of the approximations here is not sufficient to calculate the Planck scale reliably and verify our intuitive picture. Nevertheless, the picture which emerges is analogous to the low energy limit of the RS2 case, discussed in [8, 16]. There 4D evolution arose in the limit $\rho \ll \lambda$, where the Planck brane simply picked the worldvolume trajectory in AdS whose intrinsic geometry responded to the 4D matter contents of the universe. A similar situation may occur in the case of the origami in the leading order of the expansion in ρ/λ, and should be verified by performing a general analysis, for example along the lines of the derivation of the effective 4D Einstein's equations in RS2 [23].

4. Fluttering origami

Our expectation that the origami (2.14), (2.22) do admit effective 4D picture with normal long range gravity is supported by the perturbative analysis of the graviton spectrum. We consider explicitly the case of the Minkowski origami (2.14) which is simpler. The extension to the de Sitter origami (2.22) is straightforward albeit technically involved and will not be presented here. We look for the tensor perturbations of (2.14) of the form

$$g_{\mu\nu}(x^\lambda, z_k) = g^0_{\mu\nu}(z_k) + h_{\mu\nu}(x^\lambda, z_k) = \Omega^2(z)\left(\eta_{\mu\nu} + \bar{h}_{\mu\nu}(x^\lambda, z_k)\right), \tag{4.1}$$

in the transverse-traceless gauge $\partial_\mu \bar{h}^\mu{}_\nu = \bar{h}^\mu{}_\mu = 0$, where the conformal factor for the Minkowski origami (2.14) is $\Omega(z) = L/(\sum_{k=1}^2 \mathcal{C}_k|z_k| + L)$. It is convenient to define the graviton wavefunctions Ψ by $\bar{h}_{\mu\nu} = \Psi\epsilon_{\mu\nu}$, where $\epsilon_{\mu\nu}$ is the standard constant polarization tensor. Linearizing the field equations (2.2) for the variable Ψ yields the particularly simple field equation for these modes

$$\nabla^2{}_6\Psi = 0\,, \tag{4.2}$$

where $\nabla^2{}_6$ is the 6D covariant d'alembertian of (2.14), $\nabla^2{}_6 = [1/\sqrt{g_6}]\partial_A(\sqrt{g_6}g^{AB}\partial_B)$. This can be put in the familiar form of the Schrödinger equation for the graviton modes by using the conformal metric $\bar{g}_{AB} = g_{AB}/\Omega^2$, splitting it as a direct sum of the flat Minkowski metric and the 2D transverse metric and defining the wavefunction $\psi = \Omega^2\Psi$. Looking for the solutions of (4.2) in the form $\psi(x^\lambda, z_k) = \psi(z_k)\exp(ip\cdot x)$, where p^μ is the longitudinal 4-momentum, obeying $p^2 = m^2$ and m is the 4D mass of the mode, we find the Schrödinger equation for the graviton modes:

$$\bar{\Delta}_2\psi + \left(m^2 - V(z_k)\right)\psi = 0\,, \tag{4.3}$$

where the potential is given by $V(z_k) = \frac{\bar{\Delta}_2\Omega^2}{\Omega^2} = 2(\frac{\bar{\Delta}_2\Omega}{\Omega} + \frac{(\bar{\nabla}_2\Omega)^2}{\Omega^2})$, i.e. explicitly by

$$V(z_k) = \frac{6}{(\sum_{k=1}^2 \mathcal{C}_k|z_k| + L)^2} - \frac{\kappa_6^2 L}{2}\sum_{k=1}^{2}\frac{\sigma_k}{\sum_{n=1}^2 \mathcal{C}_n|z_n| + L}\delta(z_k)\,. \tag{4.4}$$

The 2D laplacian $\bar{\Delta}_2$ is defined according to $\bar{\Delta}_2 = [1/\sqrt{\bar{g}_2}]\partial_k(\sqrt{\bar{g}_2}\bar{g}^{kl}\partial_l)$ where the 2D inverse metric $\bar{g}^{kl}$ is given in (2.24), and also $(\bar{\nabla}_2\Omega)^2 = \bar{g}^{kl}\partial_k\Omega\partial_l\Omega$ etc. The equation (4.3) reduces to the familiar Schrödinger problem in the volcano potential of [6] in the case of a Minkowski brane in AdS_5. In this case, the shape of the 2D volcano potential (4.4) resembles a tablecloth with a corner raised up, and sharp, infinitely deep drops along the edges.

The zero mode solution of (4.3), which corresponds to the 4D graviton localized on the intersection and has $m^2 = 0$, is given by $\psi_0(z_k) = \frac{1}{\mathcal{N}}\Omega^2$, i.e.,

$$\psi_0(z_k) = \frac{1}{\mathcal{N}}\frac{L^2}{(\sum_{k=1}^2 \mathcal{C}_k|z_k| + L)^2}\,. \tag{4.5}$$

This can be verified by a direct substitution of (4.5) in (4.3), but in fact follows straightforwardly from (4.2), which admits the solutions $\Psi = \frac{1}{\mathcal{N}}e^{ip\cdot x}$ when $p^2 = 0$. Clearly, the zero-mode wavefunction takes its maximal value on the 3-brane at $z_k = 0$, and decreases monotonically to zero as $z_k \to \infty$, implying that the zero mode is localized to the 3-brane at the intersection. In this equation, $\mathcal{N}$ is the normalizing factor, obtained by requiring that the norm of ψ is unity. Since the norm is defined as is usual for the Schrödinger equation,

$$\int_{\text{bulk}} d^2z\sqrt{\bar{g}_2}\;\psi^*\phi = \delta_{\psi,\phi}\,, \tag{4.6}$$

for the zero mode this yields

$$\mathcal{N}^2 = 4\int_0^\infty d^2z\sqrt{\bar{g}_2}\;\frac{L^4}{(\sum_{k=1}^2 \mathcal{C}_k|z_k| + L)^4}\,. \tag{4.7}$$

It is straightforward to evaluate the integral by defining new variables $\zeta_k = \mathcal{C}_k z_k/L$ and substituting $\sqrt{\bar{g}_2} = 1/\sin(\alpha_1+\alpha_2)$. We find $\mathcal{N}^2 = [2L^2/3](\tan\alpha_1 + \tan\alpha_2)$, and therefore

$$\psi_0(z_k) = \frac{\sqrt{3}}{\sqrt{2}(\tan\alpha_1 + \tan\alpha_2)^{1/2}}\frac{L}{(\sum_{k=1}^2 \mathcal{C}_k|z_k| + L)^2}\,. \tag{4.8}$$

From this and (2.1) it follows by integrating out the bulk that the effective 4D Planck scale and the fundamental Planck scale are related by

$$\frac{1}{\kappa_4^2} = \frac{\mathcal{N}^2}{\kappa_6^2} = \frac{2L^2}{3\kappa_6^2}\,(\tan\alpha_1 + \tan\alpha_2)\,, \tag{4.9}$$

or therefore

$$M_4{}^2 = \frac{2}{3}\,(\tan\alpha_1 + \tan\alpha_2)\; M_*{}^4 L^2\,. \tag{4.10}$$

More generally, one computes the couplings starting from the action (2.1), and considering the canonically normalized graviton $\gamma_{\mu\nu} = \bar{h}_{\mu\nu}/2\kappa_6$, which couples to the matter on the 3-brane at $z_k = 0$ via the usual dimension-5 operator

$$\mathcal{L}_{int} = \kappa_6\,\psi(0,0)\,\gamma_{\mu\nu}\,T^{\mu\nu}\,, \tag{4.11}$$

where $T^{\mu\nu}$ is the stress energy tensor localized on the 3-brane at the summit of the origami (2.14). Substituting in (4.11) the zero-mode wavefunction evaluated on the intersection, $\psi_0(0,0) = \sqrt{3}/[\sqrt{2}L(\tan\alpha_1 + \tan\alpha_2)^{1/2}]$ from (4.8), and using (4.9) we see that (4.11) is indeed the coupling of the 4D graviton with the coupling constant given by the 4D Planck scale M_4 given in (4.10). A similar argument shows that the matter localized elsewhere on the 4-branes also couples with the same coupling to the 4D graviton. Note that in the limit of 4-branes with equal tension and a 3-brane with tension $\lambda \ll \sigma_k L$, such that $\alpha_1 \simeq \alpha_2 \simeq \pi/4$ this agrees with the 4D Planck scale in [9], $M_4{}^2 = \frac{4}{3}\,M_*{}^4L^2$, and confirms our intuitive argument from the previous section, where we have derived the Planck scale in this limit from cosmological considerations. Similar conclusions remain true in the case of de Sitter origami. It also localizes 4D gravity to the intersection, and a quick way to verify this is to note that the spatial volume of the section of AdS_6 bounded by the 4-branes and the bulk horizon, whose measure is defined by the metric (2.20) at any constant time t, is finite and time-independent. Hence the 4D Planck mass in that case will also be finite.

We now turn to the massive gravitons, whose wavefunctions are given by the $m^2 \neq 0$ eigenfunctions of the Schrödinger equation (4.3). The δ-functions in the potential (4.4) can be reinterpreted by a pillbox integration technique applied to the Schrödinger equation (4.3) as boundary conditions on the normal gradients of the eigenmodes on the 4-branes. Hence the eigenmode problem defined by the Schrödinger equation (4.3) with the potential (4.4) is equivalent to the problem of finding eigenmodes on the four AdS_6 wedges between the branes, and matching them according to the boundary conditions enforced by the discontinuities on the branes. Substituting $V = \Delta_2\Omega^2/\Omega^2$ in (4.3) for arbitrary eigenmode ψ with $m^2 \neq 0$, and manipulating slightly the terms yields the identity

$$\partial_k\left[\sqrt{\bar{g}}\bar{g}^{kl}\left(\Omega^2\partial_l\psi - \psi\partial_l\Omega^2\right)\right] + \sqrt{\bar{g}}\,m^2\,\Omega^2\psi = 0\,. \tag{4.12}$$

Integrating over each z_k in the interval $(-\epsilon,\epsilon)$ and using continuity of Ω^2 and ψ and boundedness of $\bar{g}^{kl}$ gives

$$\bar{g}^{kl}\frac{\partial\psi}{\partial z_l}\Big|_{z_k=0^+} - \bar{g}^{kl}\frac{\partial\psi}{\partial z_l}\Big|_{z_k=0^-} = \bar{g}^{kl}\frac{1}{\Omega}\frac{\partial\Omega}{\partial z_l}\psi\Big|_{z_k=0}\,, \qquad k \in \{1,2\}\,, \tag{4.13}$$

which fix the jump of the derivatives of the wavefunction across the 4-branes. Here $0^{\pm}$ refers to the different sides of a 4-brane, with wavefunctions evaluated in adjacent wedges. Away from the 4-branes, the laplacian $\bar{\Delta}_2$ is given by the laplacian in each segment Δ_2, depending only on the new (old!) coordinates $\tilde{z}_k = |z_k|$ and the metric $\hat{g}^{kl} = \vec{n}_k \cdot \vec{n}_l$, $\Delta_2 = (1/\sqrt{\hat{g}})\tilde{\partial}_k(\sqrt{\hat{g}}\hat{g}^{kl}\tilde{\partial}_l)$. The potential reduces to $V = 6/(\sum_{k=1}^{2} \mathcal{C}_k \tilde{z}_k + L)^2$, and so the eigenvalue problem for the Schrödinger equation (4.3) maps on the boundary value problem on the four wedges $\tilde{z}_k \geq 0$, which after a simple algebra can be written as

$$\Delta_2 \psi + \left(m^2 - \frac{6}{(\sum_{k=1}^{2} \mathcal{C}_k \tilde{z}_k + L)^2} \right) \psi = 0 \,,$$
$$\hat{g}^{kl} \left(\frac{\partial \psi}{\partial \tilde{z}_l}\Big|_{z_k=0^+} + \frac{\partial \psi}{\partial \tilde{z}_l}\Big|_{z_k=0^-} \right) + 4 \frac{\mathcal{C}_k + \mathcal{C}_l \, \vec{n}_k \cdot \vec{n}_l}{\sum_{n=1}^{2} \mathcal{C}_n \tilde{z}_n + L} \psi \Big|_{z_k=0} = 0 \,, \qquad k \in \{1,2\} \,, \tag{4.14}$$

where the sign flips in the boundary conditions come after changing variables from z_k to $\tilde{z}_k = |z_k|$. It is easy to check that the zero mode wavefunction ψ_0 in (4.5) satisfies (4.14) identically.

To solve the boundary value problem (4.14), note that inside each wedge $\tilde{z}_k \geq 0$ the potential depends only on the radial coordinate in the Poincare patch of AdS_6. It is independent of the coordinate parallel with the AdS_6 boundary. Thus we can separate variables by going back to the coordinates w_1, w_2 defined by (2.11) (see figure 2) and substituting $\psi = \phi(w_1) \exp(iqw_2)$, where we take $0 < q^2 < m^2$, for reasons to be explained below. This reduces (4.14) to an ordinary differential equation for ϕ,

$$\frac{d^2\phi}{dw_1{}^2} + \left(m^2 - q^2 - \frac{6}{(w_1 + L)^2} \right) \phi = 0 \,, \tag{4.15}$$

which, for a given mass m and a transverse momentum q, upon defining the new variable $\rho = \mu(w_1 + L)$, where $\mu = \sqrt{m^2 - q^2} > 0$, and substituting $\phi = \sqrt{\rho}\chi$, we recognize as the Bessel differential equation

$$\frac{d^2\chi}{d\rho^2} + \frac{1}{\rho}\frac{d\chi}{d\rho} + \left(1 - \frac{25}{4\rho^2} \right) \chi = 0 \,. \tag{4.16}$$

The solutions of this equation are Bessel functions $J_{\pm 5/2}(\rho)$, which are linearly independent because their index is half-integer. In fact, they can be written in closed form, and it is convenient to define the functions $\phi_\pm(\rho) = \sqrt{\rho} J_{\pm 5/2}(\rho)$, which are

$$\phi_+(\rho) = \sqrt{\frac{2}{\pi}} \left(3 \frac{\sin\rho}{\rho^2} - 3 \frac{\cos\rho}{\rho} - \sin\rho \right) \,,$$
$$\phi_-(\rho) = \sqrt{\frac{2}{\pi}} \left(3 \frac{\cos\rho}{\rho^2} + 3 \frac{\sin\rho}{\rho} - \cos\rho \right) \,. \tag{4.17}$$

General solutions of (4.14) are given by linear combinations of functions of the form $\exp(\pm iqw_2)\phi_\pm[\mu(w_1 + L)]$ chosen to satisfy the boundary conditions for normal derivatives at $\tilde{z}_k = 0$, given in (4.14). These wavefunctions should be at least δ-function normalizable. To ensure this we must restrict q^2, m^2 to obey the ordering relation $0 < q^2 < m^2$. For q^2

and m^2 in conflict with this relation, the wavefunctions would diverge in the limit $w_1 \to \infty$, and would not be normalizable even to a δ-function. Hence the spectrum of bulk gravitons is bounded from below, with the zero mode ψ_0 in (4.8) being the minimum mass state, and so there are no unstable, runaway modes.

The solutions $\exp(\pm iqw_2)\phi_\pm[\mu(w_1+L)]$ with $0 < q^2 < m^2$ are continuously degenerate, however in general the functions with a fixed m and q are not orthogonal to each other. Thus we need to determine those linear combinations which satisfy the boundary conditions in (4.14) and are orthogonal. By the linearity of the boundary conditions, it is sufficient to consider only the real functions ψ. After a simple algebra, we find that a general solution of the Schrödinger equation with a fixed m and q^2 inside a wedge between the 4-branes is parameterized by two real numbers $\mathcal{A}(q)$ and $\mathcal{B}(q)$ and two phases $\theta(q), \vartheta(q)$,

$$\psi_m = \mathcal{A}(q)\cos(qw_2 - \theta(q))\,\phi_+[\mu(w_1+L)] + \mathcal{B}(q)\cos(qw_2 - \vartheta(q))\,\phi_-[\mu(w_1+L)]\,. \quad (4.18)$$

A complete wavefunction for a given m and q^2 would then be specified by four such expressions, one for each wedge between the 4-branes. However: wavefunctions with fixed m and q^2 cannot satisfy the boundary conditions in (4.14), unless of course both 4-branes are parallel with the AdS_6 boundary (in which case there is no localized 4D gravity and no effective low energy 4D theory). The reason is that the 4-branes break the translational invariance of the bulk in the w_2 direction, and hence the scattering of bulk waves on the 4-branes does not conserve the momentum in the w_2 direction, q. Intuitively, because the boundary conditions in (4.14) can be treated as the δ-function terms in the potential (4.4), the procedure which we employ, i.e. solving the mode equation away from the 4-branes by the separation of variables yielding (4.18) and then imposing the boundary conditions in (4.14), is completely equivalent to splitting the hamiltonian associated with (4.4) into the leading order term, controlled by the bulk potential, and a perturbation, given by the δ-functions. Since they are invariant under different symmetries, in general they do not commute, and so the eigenvalues correspond to the subset of wavefunctions which are annihilated by the commutator of these two operators. Thus an eigenmode of (4.14) for a given eigenvalue m^2 is a *linear superposition*[4] of the functions (4.18) with any q obeying $0 < q^2 < m^2$:

$$\psi_m = \int_0^m dq\,\left(\mathcal{A}(q)\cos(qw_2 - \theta(q))\,\phi_+[\mu(w_1+L)] + \mathcal{B}(q)\cos(qw_2 - \vartheta(q))\,\phi_-[\mu(w_1+L)]\right). \quad (4.19)$$

We should pick it so that it satisfies the boundary conditions in (4.14).

Determining the eigenmodes (4.19) is more tractable when the origami (2.14) is orbifolded by the largest possible discrete symmetry group. This happens for the symmetric origami built out of the 4-branes with identical tensions, and cutting the AdS_6 bulk at identical angles: $\sigma_1 = \sigma_2 = \sigma$ and so $\alpha_1 = \alpha_2 = \alpha$. In light of our intuitive argument on

[4] The authors of [24] looked for the bulk KK eigenmodes in the case of special intersections with $\alpha_1 = \alpha_2 = \pi/4$ and tensionless 3-brane. They did not succeed in finding these modes because they only sought for them as functions of fixed m and q instead of as superpositions of modes with q in the allowed range $0 < q^2 < m^2$.

how to restore the correct 4D cosmology, this is the interesting phenomenological limit. In this case, the discrete symmetries of the background (2.14) are the reflections $z_k \leftrightarrow -z_k$ and the four rotations about the intersection by angle α. Identifying by the rotations reduces the four wedges to a single one, implying that the wavefunction (4.19) is given by the same formula in every segment between the 4-branes. In addition, orbifolding by the reflections implies that the wavefunction depends only on $|z_k|$'s. Together, the symmetries further impose the condition that the wavefunction must be extremized everywhere along the radial AdS_6 direction, $w_2 = 0$, because it must be symmetric under the permutation $z_1 \leftrightarrow z_2$. This simply follows from the fact that the four rotations and the reflections complete the full permutation group of four elements. Thus we must pick $\theta(q) = \vartheta(q) = 0$ in (4.19). The ansätz (4.19) becomes

$$\psi_m = \int_0^m dq\, \cos(q w_2)\, \left(\mathcal{A}(q)\, \phi_+[\mu(w_1+L)] + \mathcal{B}(q)\, \phi_-[\mu(w_1+L)]\right). \tag{4.20}$$

The coordinates w_k are related to $\tilde{z}_k = |z_k|$ according to $w_1 = \frac{|z_1|+|z_2|}{2\sin\alpha}$, $w_2 = \frac{|z_2|-|z_1|}{2\cos\alpha}$, and because of the symmetries the boundary conditions reduce to a single functional identity, at say $z_1 = 0$:

$$\frac{\partial\psi_m}{\partial w_1}\Big|_{z_1=0} - \cot\alpha\, \frac{\partial\psi_m}{\partial w_2}\Big|_{z_1=0} + \frac{2}{w_1+L}\psi_m\Big|_{z_1=0} = 0\,. \tag{4.21}$$

Upon substituting (4.20) into this equation, defining dimensionless variables $x = \mu/m$, $y = mz_2/(2\sin\alpha)$ and $l = mL$, changing the integration variable to x by using $q = \sqrt{1-x^2}m$, and assuming analyticity of $\mathcal{A}$ and $\mathcal{B}$ in the interval $0 < q^2 < m^2$ so that they depend on q only through x, we obtain an integral equation for the functions $\mathcal{A}(x)$ and $\mathcal{B}(x)$:

$$\begin{aligned}\int_0^1 dx\, x\, \Big\{\mathcal{A}(x)\Big[\Big(3\frac{\sin[x(y+l)]}{x^2(y+l)^2} - 3\frac{\cos[x(y+l)]}{x(y+l)} - \sin[x(y+l)]\Big)\sin(\sqrt{1-x^2}y\tan\alpha) + \\ + \frac{x\tan\alpha}{\sqrt{1-x^2}}\Big(\frac{\sin[x(y+l)]}{x(y+l)} - \cos[x(y+l)]\Big)\cos(\sqrt{1-x^2}y\tan\alpha)\Big] + \\ + \mathcal{B}(x)\Big[\Big(3\frac{\cos[x(y+l)]}{x^2(y+l)^2} + 3\frac{\sin[x(y+l)]}{x(y+l)} - \cos[x(y+l)]\Big)\sin(\sqrt{1-x^2}y\tan\alpha) + \\ + \frac{x\tan\alpha}{\sqrt{1-x^2}}\Big(\frac{\cos[x(y+l)]}{x(y+l)} + \sin[x(y+l)]\Big)\cos(\sqrt{1-x^2}y\tan\alpha)\Big]\Big\} = 0\,.\end{aligned} \tag{4.22}$$

This equation should be viewed as a functional identity. It says that the left-hand side, which is an analytic function of y's in order to ensure that the massive KK modes (4.20) are plane wave-normalizable, must vanish for all values of y. This is equivalent to saying that all y-derivatives of the left-hand side should vanish at $y = 0$.

To see why (4.22) should have solutions, we return to the boundary condition (4.21), and reinterpret it in terms of the normal derivatives as follows. First, we change the coordinates back to the Poincare patch centered on the AdS boundary as in (2.4), which would replace $w_1 + L$ in (4.21) by w_1. Then by going to the polar coordinates, we can see that (4.21), at $z_1 = 0$, becomes exactly $\frac{\partial\psi}{\partial\theta} = -2\tan\alpha\,\psi$, where θ is the polar angle in the Poincare patch. Thus we see that (4.21) amounts to fixing the logarithmic gradient of the wavefunction to be $-2\tan\alpha$ on the 4-branes. Along with the ansätz (4.20) which requires that the derivatives of the wavefunction vanish along the radial direction in AdS_6, this fully determines the boundary value problem. We should find its solutions by an appropriate convolution of wavefunctions with various values of q. From this argument, we can understand the integral equation (4.22) as the requirement that an eigenmode of a given mass m which is extremized along the radial direction of AdS_6 has a vanishing overlap with all wavefunctions with the logarithmic gradient different from $-2\tan\alpha$. The number of solutions with a mass m increases with m, asymptotically approaching a linear function of m as the mass exceeds $1/L$, to match the degeneracy of states which inhabit two extra dimensions.

The explicit form of the eigenmodes is not needed to deduce their couplings and forces which they mediate; we can estimate them from (4.22) by looking only at the values of the wavefunction and its first derivative on the intersection. For light modes, $m \ll 1/L$ or therefore $l \ll 1$, the solutions must behave as

$$\begin{aligned} \mathcal{A}(x) &\simeq \frac{1}{\mathcal{N}_m}\,, \\ \mathcal{B}(x) &\simeq \frac{xl^3}{\mathcal{N}_m}\,, \end{aligned} \tag{4.23}$$

in order to remain analytic and satisfy (4.22) to the leading order at small y. At large radial distances, the normalization (4.6) requires that $\mathcal{N}_m \simeq \mathcal{N}_0\sqrt{\tan\alpha}$ for (4.20) to be δ-function normalizable, where $\mathcal{N}_0$ is a number of order unity. The coupling of these modes to the matter on the intersection is determined by the same operator as the coupling of the zero-mode, (4.11). Substituting $\psi_m(0,0) \simeq \frac{ml}{\mathcal{N}_0\sqrt{\tan\alpha}} \simeq \frac{m^2L}{\mathcal{N}_0\sqrt{\tan\alpha}}$ in (4.11) we find the coupling constant of the light KK modes to $T^{\mu\nu}$ at the tip of the origami:

$$g_{m\ll 1/L} \simeq \frac{1}{\bar{\mathcal{N}}}\frac{m^2L^2}{M_4}\,, \tag{4.24}$$

where $\bar{\mathcal{N}}$ is a constant of order unity. A similar argument shows that for the modes much heavier than the inverse AdS radius, $m \gg 1/L$, there is no tunnelling suppression of the coupling since they easily fly over the AdS barrier, and so

$$g_{m\gg 1/L} \simeq \frac{1}{\bar{\mathcal{N}}M_4}\,. \tag{4.25}$$

We note that the suppression effects in the couplings are the 6D generalization of the tunnelling suppression in AdS_5, studied in detail in [18].

Having found the couplings, we can estimate the newtonian potential between two point masses on the intersection. The KK modes contribute with a Yukawa suppression coming from their masses, and with the tunnelling suppression for the light modes. Squaring the couplings, we can approximate the potential with

$$V = -G_N\frac{m_1m_2}{r}\left(1 + \mathbf{a}\int_0^{1/L} dmL\,n(m)\,(mL)^4\,e^{-mr} + \mathbf{b}\int_{1/L}^{M_*} dmL\,n(m)\,e^{-mr}\right), \tag{4.26}$$

where the first term in the bracket comes from the zero mode, the second from the light modes $m \ll 1/L$ and the last from the heavy modes, $m \gg 1/L$, the coefficients $\mathbf{a}$ and $\mathbf{b}$ are numbers of order unity, and the Newton's constant is $G_N = 8\pi/{M_4}^2$. The density of states

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$n(m)$ asymptotically approaches a linear function of m. Even if we ignore the detailed form of the function $n(m)$ we can see that the corrections to the newtonian potential are small; the integrals in (4.26) at distances $r \gg L$ then give

$$V = -G_N \frac{m_1 m_2}{r} \left(1 + \mathbf{c} \left(\frac{L}{r}\right)^5 + \sum_{k=1}^{5} \mathbf{c}_k \left(\frac{L}{r}\right)^k e^{-r/L} + \cdots\right), \tag{4.27}$$

where again $\mathbf{c}, \mathbf{c}_k$ are numbers of order unity. The leading term is the 4D newtonian potential, generated by the exchange of the zero-mode graviton. The second term comes from the lightest end of the continuum, which is long-range but suppressed by at least five extra powers of the distance, $\delta V \simeq -G_N \mathbf{c} \frac{m_1 m_2 L^5}{r^6}$, because of the tunnelling suppression. Thus when $r \gg L$ this is the leading correction because other terms are exponentially suppressed, and so the corrections are extremely weak. As r decreases below L, the contributions from the light modes that are weighed by higher powers of L/r cancel out at short distances. This can be seen by evaluating the integral for the light modes and expanding the exponentials, for any analytic function $n(m)$. In this limit the leading correction to the Newton's potential in (4.27) arises from a term coming from the second of the integrals in (4.26), that corresponds to the contributions of the modes with masses $m \sim 1/L$, whose couplings are not tunnelling-suppressed. Thus the dominant correction in (4.26) in the limit $r \ll L$ behaves as $\delta V \sim -G_N \frac{m_1 m_2 L^2}{r^3} \sim -\frac{1}{M_*{}^4} \frac{m_1 m_2}{r^3}$, due to the multiplicity of heavy states. In the intermediate regime $r \sim L$, the corrections are approximated by $\delta V \sim -G_N \frac{m_1 m_2 L}{r^2}$. The present bounds on the corrections to Newton's force from tabletop experiments then yield a bound on the AdS radius, $L \leq 10^{-4}$m [25]. As a consequence, the long distance gravitational interactions between objects on the intersection are indeed governed by the 4D Newton's force with a great accuracy. For a more precise determination of the potential a calculation along the lines of [26, 27] is needed.

The formula for the 4D Planck mass (4.10) together with the bound on the AdS radius suggests an interesting phenomenological possibility. For symmetric origami, (4.10) reduces to $M_4{}^2 = \frac{4}{3} \tan\alpha \, M_*{}^4 L^2$. Therefore even if the AdS radius is as large as $L \sim 10^{-4}$m, the fundamental scale can be $M_* \sim$ few $\times$ TeV, still yielding the correct value of the 4D Planck scale, $M_4 \simeq 10^{19} GeV$, if the total angle between the branes is less than $\pi/2$, so that $\tan\alpha < 1$. This could help to relax some of the astrophysical bounds, which constrain models with large extra dimensions, in the origami case [1, 28]. Scenarios where the Planck-electroweak hierarchy arises partially because of the shape and topology of the compactifications were also discussed in [29, 30]. It would be therefore interesting to explore if such a scenario is viable, and consistent with the low energy limits needed to reproduce consistent 4D cosmology, along the lines discussed in the previous section.

5. Summary

In this paper we have derived exact solutions describing the intersection of two 4-branes in AdS_6 on a 3-brane of arbitrary non-negative tension. These solutions correspond to the vacua of the theory on the intersection, and generalize the analysis for the RS2 case of [8]. In the case of the Minkowski and de Sitter origami, they localize 4D graviton on the intersection, yielding a fully consistent 4D effective theory in the low energy limit. We have explicitly computed the zero-mode wavefunction, and couplings of KK continuum, and evaluated the gravitational potential between two masses localized on the intersection. In the case of the *AdS* origami there is no 4D localized graviton, but they may be an interesting arena to study the phenomenon of quasilocalization of [20].

The Minkowski and de Sitter origami which localize 4D gravity to the intersection may be a natural background to formulate the low energy cosmology on codimension-2 braneworlds. We have elucidated the right limit where the homogeneous cosmological perturbations on the intersection gravitate like normal matter in 4D universe governed by the usual 4D Einstein gravity, as long as the order of scales $\rho \ll \lambda \ll \sigma_k$ is maintained. We stress that the current precision of cosmological data allows for the inequalities to be satisfied by roughly two orders of magnitude, without spoiling the existing bounds on the validity of Einstein gravity. This makes the inequalities rather easy to satisfy, and opens up a possibility of developing models where such limits could be natural future attractors of cosmological evolution. In this context, one should also ask questions about the stabilization of the shape moduli, which are the angles between the 4-branes, which are fixed once the tensions are assigned. In the early universe, when the origami is perturbed, the shape moduli could roll, and it would be of interest to find out the details of their dynamics. We hope to return to these questions elsewhere.

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References

[1] N. Arkani-Hamed, S. Dimopoulos and G.R. Dvali, *The hierarchy problem and new dimensions at a millimeter*, *Phys. Lett.* **B 429** (1998) 263 [hep-ph/9803315]; *Phenomenology, astrophysics and cosmology of theories with sub-millimeter dimensions and TeV scale quantum gravity*, *Phys. Rev.* **D 59** (1999) 086004 [hep-ph/9807344]; I. Antoniadis, N. Arkani-Hamed, S. Dimopoulos and G.R. Dvali, *New dimensions at a millimeter to a fermi and superstrings at a TeV*, *Phys. Lett.* **B 436** (1998) 257 [hep-ph/9804398].

[2] I. Antoniadis, *A possible new dimension at a few TeV*, *Phys. Lett.* **B 246** (1990) 377; J.D. Lykken, *Weak scale superstrings*, *Phys. Rev.* **D 54** (1996) 3693 [hep-th/9603133].

[3] P. Hořava and E. Witten, *Heterotic and type-I string dynamics from eleven dimensions*, *Nucl. Phys.* **B 460** (1996) 506 [hep-th/9510209]; *Eleven-dimensional supergravity on a manifold with boundary*, *Nucl. Phys.* **B 475** (1996) 94 [hep-th/9603142]; E. Witten, *Strong coupling expansion of Calabi-Yau compactification*, *Nucl. Phys.* **B 471** (1996) 135 [hep-th/9602070].

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[4] L. Randall and R. Sundrum, *A large mass hierarchy from a small extra dimension*, *Phys. Rev. Lett.* **83** (1999) 3370 [hep-ph/9905221].

[5] J.M. Maldacena, *The large-N limit of superconformal field theories and supergravity*, *Adv. Theor. Math. Phys.* **2** (1998) 231 [hep-th/9711200];
S.S. Gubser, I.R. Klebanov and A.M. Polyakov, *Gauge theory correlators from non-critical string theory*, *Phys. Lett.* **B 428** (1998) 105 [hep-th/9802109];
E. Witten, *Anti-de Sitter space and holography*, *Adv. Theor. Math. Phys.* **2** (1998) 253 [hep-th/9802150].

[6] L. Randall and R. Sundrum, *An alternative to compactification*, *Phys. Rev. Lett.* **83** (1999) 4690 [hep-th/9906064].

[7] J. Lykken and L. Randall, *The shape of gravity*, *J. High Energy Phys.* **06** (2000) 014 [hep-th/9908076].

[8] N. Kaloper, *Bent domain walls as braneworlds*, *Phys. Rev.* **D 60** (1999) 123506 [hep-th/9905210].

[9] N. Arkani-Hamed, S. Dimopoulos, G.R. Dvali and N. Kaloper, *Infinitely large new dimensions*, *Phys. Rev. Lett.* **84** (2000) 586 [hep-th/9907209].

[10] C. Csáki and Y. Shirman, *Brane junctions in the Randall-Sundrum scenario*, *Phys. Rev.* **D 61** (2000) 024008 [hep-th/9908186];
A.E. Nelson, *A new angle on intersecting branes in infinite extra dimensions*, *Phys. Rev.* **D 63** (2001) 087503 [hep-th/9909001];
J. Jiang and T.J. Li, *Self-tuning and de Sitter brane intersections in the 6-dimensional brane models*, *Eur. Phys. J.* **C 26** (2003) 473 [hep-ph/0109073].

[11] N. Kaloper and A.D. Linde, *Inflation and large internal dimensions*, *Phys. Rev.* **D 59** (1999) 101303 [hep-th/9811141].

[12] H.A. Chamblin and H.S. Reall, *Dynamic dilatonic domain walls*, *Nucl. Phys.* **B 562** (1999) 133 [hep-th/9903225].

[13] T. Nihei, *Inflation in the five-dimensional universe with an orbifold extra dimension*, *Phys. Lett.* **B 465** (1999) 81 [hep-ph/9905487].

[14] P. Binetruy, C. Deffayet and D. Langlois, *Non-conventional cosmology from a brane-universe*, *Nucl. Phys.* **B 565** (2000) 269 [hep-th/9905012];
P. Binetruy, C. Deffayet, U. Ellwanger and D. Langlois, *Brane cosmological evolution in a bulk with cosmological constant*, *Phys. Lett.* **B 477** (2000) 285 [hep-th/9910219].

[15] S.W. Hawking, T. Hertog and H.S. Reall, *Brane new world*, *Phys. Rev.* **D 62** (2000) 043501 [hep-th/0003052];
J. Garriga and M. Sasaki, *Brane-world creation and black holes*, *Phys. Rev.* **D 62** (2000) 043523 [hep-th/9912118].

[16] C. Csáki, M. Graesser, C.F. Kolda and J. Terning, *Cosmology of one extra dimension with localized gravity*, *Phys. Lett.* **B 462** (1999) 34 [hep-ph/9906513];
C. Csáki, M. Graesser, L. Randall and J. Terning, *Cosmology of brane models with radion stabilization*, *Phys. Rev.* **D 62** (2000) 045015 [hep-ph/9911406];
J.M. Cline, C. Grojean and G. Servant, *Cosmological expansion in the presence of extra dimensions*, *Phys. Rev. Lett.* **83** (1999) 4245 [hep-ph/9906523];

JHEP05(2004)061

H.B. Kim and H.D. Kim, *Inflation and gauge hierarchy in randall-sundrum compactification*, *Phys. Rev.* **D 61** (2000) 064003 [hep-th/9909053];
P. Kraus, *Dynamics of anti-de Sitter domain walls*, *J. High Energy Phys.* **12** (1999) 011 [hep-th/9910149].

[17] N. Kaloper, *Crystal manyfold universes in AdS space*, *Phys. Lett.* **B 474** (2000) 269 [hep-th/9912125];
S.-k. Nam, *Modeling a network of brane worlds*, *J. High Energy Phys.* **03** (2000) 005 [hep-th/9911104];
I.I. Kogan, S. Mouslopoulos, A. Papazoglou and G.G. Ross, *Multi-brane worlds and modification of gravity at large scales*, *Nucl. Phys.* **B 595** (2001) 225 [hep-th/0006030];
J. Jiang, T.J. Li and D. Marfatia, *Brane networks in AdS space*, *Phys. Lett.* **B 492** (2000) 187 [hep-th/0007039];
S. Corley and D.A. Lowe, *Solving the hierarchy problem with brane crystals*, *Phys. Lett.* **B 505** (2001) 197 [hep-ph/0101021].

[18] S. Dimopoulos, S. Kachru, N. Kaloper, A.E. Lawrence and E. Silverstein, *Small numbers from tunneling between brane throats*, *Phys. Rev.* **D 64** (2001) 121702 [hep-th/0104239];
Generating small numbers by tunneling in multi-throat compactifications, hep-th/0106128;
N. Kaloper, *Small numbers from tunneling between brane throats*, *Prog. Theor. Phys. Suppl.* **148** (2003) 29.

[19] G. Hayward, *Gravitational action for space-times with nonsmooth boundaries*, *Phys. Rev.* **D 47** (1993) 3275.

[20] A. Karch and L. Randall, *Locally localized gravity*, *J. High Energy Phys.* **05** (2001) 008 [hep-th/0011156].

[21] J. Cline, C. Grojean and G. Servant, *Inflating intersecting branes and remarks on the hierarchy problem*, *Phys. Lett.* **B 472** (2000) 302 [hep-ph/9909496].

[22] J.M. Cline, J. Descheneau, M. Giovannini and J. Vinet, *Cosmology of codimension-two braneworlds*, *J. High Energy Phys.* **06** (2003) 048 [hep-th/0304147];
P. Bostock, R. Gregory, I. Navarro and J. Santiago, *Einstein gravity on the codimension 2 brane?*, hep-th/0311074;
H.M. Lee and G. Tasinato, *Cosmology of intersecting brane world models in gauss- bonnet gravity*, *JCAP* **04** (2004) 009 [hep-th/0401221].

[23] T. Shiromizu, K.-i. Maeda and M. Sasaki, *The Einstein equations on the 3-brane world*, *Phys. Rev.* **D 62** (2000) 024012 [gr-qc/9910076].

[24] J.E. Kim, B. Kyae and H.M. Lee, *Localized gravity and mass hierarchy in $D = 6$ with Gauss-Bonnet term*, *Phys. Rev.* **D 64** (2001) 065011 [hep-th/0104150].

[25] C.D. Hoyle et al., *Sub-millimeter tests of the gravitational inverse-square law: a search for 'large' extra dimensions*, *Phys. Rev. Lett.* **86** (2001) 1418 [hep-ph/0011014];
EOT-WASH Group collaboration, E.G. Adelberger, *Sub-millimeter tests of the gravitational inverse square law*, hep-ex/0202008;
E.G. Adelberger, B.R. Heckel and A.E. Nelson, *Tests of the gravitational inverse-square law*, *Ann. Rev. Nucl. Part. Sci.* **53** (2003) 77 [hep-ph/0307284].

[26] J. Garriga and T. Tanaka, *Gravity in the brane-world*, *Phys. Rev. Lett.* **84** (2000) 2778 [hep-th/9911055].

[27] S.B. Giddings, E. Katz and L. Randall, *Linearized gravity in brane backgrounds*, *J. High Energy Phys.* **03** (2000) 023 [hep-th/0002091].

[28] S. Cullen and M. Perelstein, *SN1987A constraints on large compact dimensions*, *Phys. Rev. Lett.* **83** (1999) 268 [hep-ph/9903422];
L.J. Hall and D.R. Smith, *Cosmological constraints on theories with large extra dimensions*, *Phys. Rev.* **D 60** (1999) 085008 [hep-ph/9904267];
V.D. Barger, T. Han, C. Kao and R.J. Zhang, *Astrophysical constraints on large extra dimensions*, *Phys. Lett.* **B 461** (1999) 34 [hep-ph/9905474];
S. Hannestad and G. Raffelt, *New supernova limit on large extra dimensions*, *Phys. Rev. Lett.* **87** (2001) 051301 [hep-ph/0103201]; *Stringent neutron-star limits on large extra dimensions*, *Phys. Rev. Lett.* **88** (2002) 071301 [hep-ph/0110067].

[29] N. Kaloper, J. March-Russell, G.D. Starkman and M. Trodden, *Compact hyperbolic extra dimensions: branes, Kaluza-Klein modes and cosmology*, *Phys. Rev. Lett.* **85** (2000) 928 [hep-ph/0002001].

[30] K.R. Dienes, *Shape versus volume: making large flat extra dimensions invisible*, *Phys. Rev. Lett.* **88** (2002) 011601 [hep-ph/0108115];
K.R. Dienes and A. Mafi, *Shadows of the planck scale: the changing face of compactification geometry*, *Phys. Rev. Lett.* **88** (2002) 111602 [hep-th/0111264].

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